Chapter 7

Advanced Network Analyses & Scheduling

Project Management for Business, Engineering, and Technology

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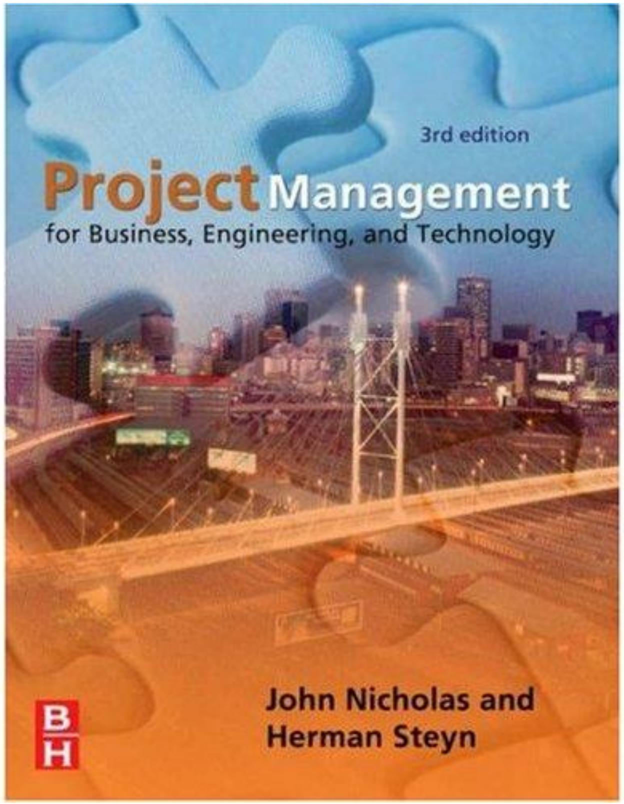


Advanced Network Analyses & Scheduling

- In Chapter 6 we assumed that activity durations are known and fixed while in practice they are merely estimates and thus variable

- This chapter addresses variability of activity duration, tradeoffs between cost and time, as well as ways to reduce project duration

Advanced Network Analyses & Scheduling

- What happens when a project takes too long (i.e., estimated completion time exceeds required target date)?
- How do we shorten project duration?
- By how much can project duration be shortened?

Time – Cost Tradeoff

The house built in less than 4 hours

Time – Cost Tradeoff

The house built in less than 4 hours

Specifications:

- Four bedrooms
- Pre-fabricated wall panels on established foundation
- Wooden floor, roof (steel), ceilings, decks & steps
- Doors, windows, a bath, toilet, plumbing & electrical
- Painted walls, ceilings & window frames
- Carpets & curtains
- Front path & a letter box
- Clothes line, wooden fence
- 3 trees planted, lawns leveled and grassed.

Time – Cost Tradeoff

The house built in less than 4 hours

External walls being constructed

Time – Cost Tradeoff

The house built in less than 4 hours

Internal walls & kitchen cabinets under construction

Time – Cost Tradeoff (cont'd)

How do we reduce project duration?

- Reduce amount of work (scope, requirements)
- Use appropriate technology
- Pay attention to motivation of project team
- Get support from executives and other stakeholders
- Use more resources
- Use more sophisticated / expensive resource(s)

Time – Cost Tradeoff (cont'd)

- In certain cases, activity duration is fixed and more resources won't help reduce duration
  - Examples: …
- In other cases, activity duration can be reduced by using more or better resources
  - Examples:…
  - Better / more resources normally cost more (activity duration is a function of cost)

Time – Cost Tradeoff (cont'd)

- The method to trade off time and cost is called "Critical Path Method" or CPM (although general methods to determine float are sometimes also referred to as CPM)
- Developed in the chemical industry in 1957 for DuPont Company (chemical plant construction)

Time – Cost Tradeoff (cont'd)

Where the use of more costly resources would reduce duration

- **In some cases**, two conditions only. Examples:
  - Either walk or take a cab
  - Chose between going by car or by airplane

The two conditions:

  - "normal" alternative – walk
  - "crash" alternative – take cab (more costly but faster)

- **In other cases**, in-between possibilities do exist (E.g. walk half the way and take cab for the other half)

Time – Cost Tradeoff (cont'd)

Cases where activity duration can be reduced by using more or better resources

- Use more (e.g. temporary) people
- Make use of overtime at a higher hourly cost
- Use high technology instead of low technology
- Ground moving equipment instead of manual labor
- …

Time – Cost Tradeoff (cont'd)

The points of "normal" and "crash" are the extreme alternatives:

- Normal: whatever work effort is considered "normal"
- Crash: maximum resources applied to obtain shortest time
- Normal effort is least costly
- Crash effort is most costly

Time – Cost Tradeoff (cont'd)

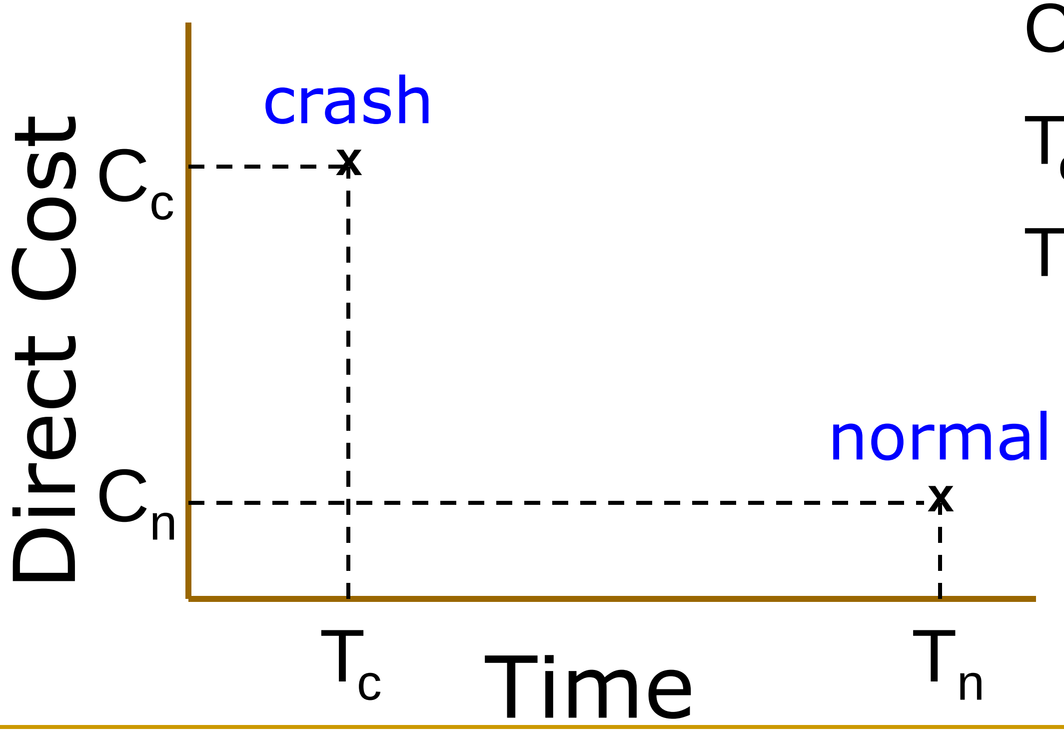
Where the use of more costly resources would reduce duration

Both the *Normal time* (T_n) and *Crash time* (Tc) are fixed values:

In its simplest form, the CPM time-cost tradeoff does not take variability (other than that relating to cost) into account

Time – Cost Tradeoff (cont'd)

Consider activity durations and direct costs for "Normal" and "Crash" conditions



C_c = Crash cost

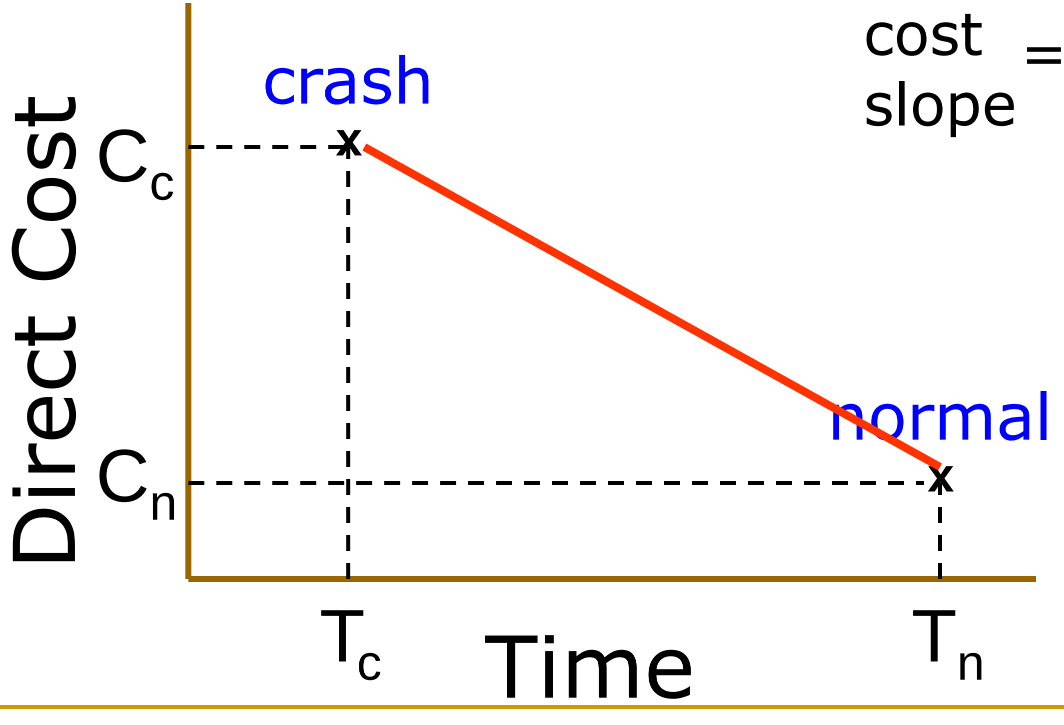
C_n = Normal cost

T_c = Crash time

T_n = Normal time

Time – Cost Tradeoff (cont'd)

Where in-between possibilities exist, assume a straight line (not stepwise, concave, etc)



$$\text{cost slope} = \frac{\Delta C}{\Delta T} = \frac{C_c - C_n}{T_c - T_n}$$

Time – Cost Tradeoff (cont'd)

- Cost slope is marginal change in cost per unit change in time
- Slope is negative since cost increases as time decreases, and vice versa
- We will ignore the negative sign

Time – Cost Tradeoff (cont'd)

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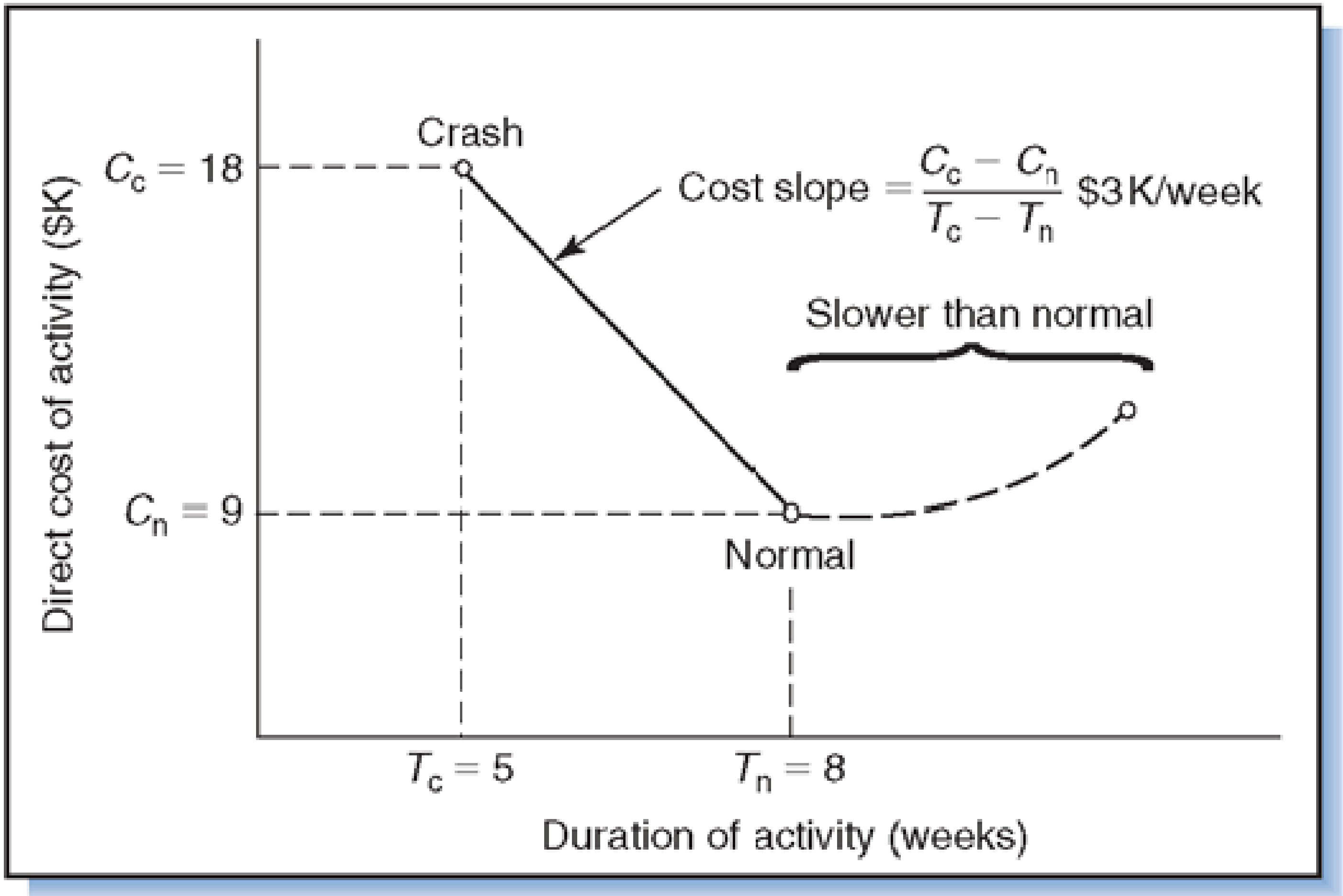
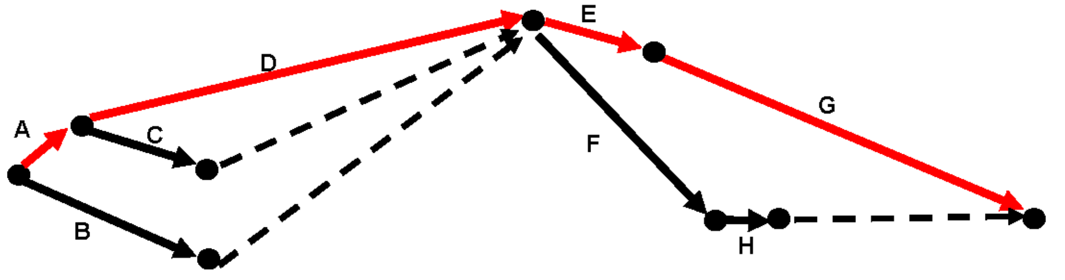


Time – Cost Tradeoff (cont'd)

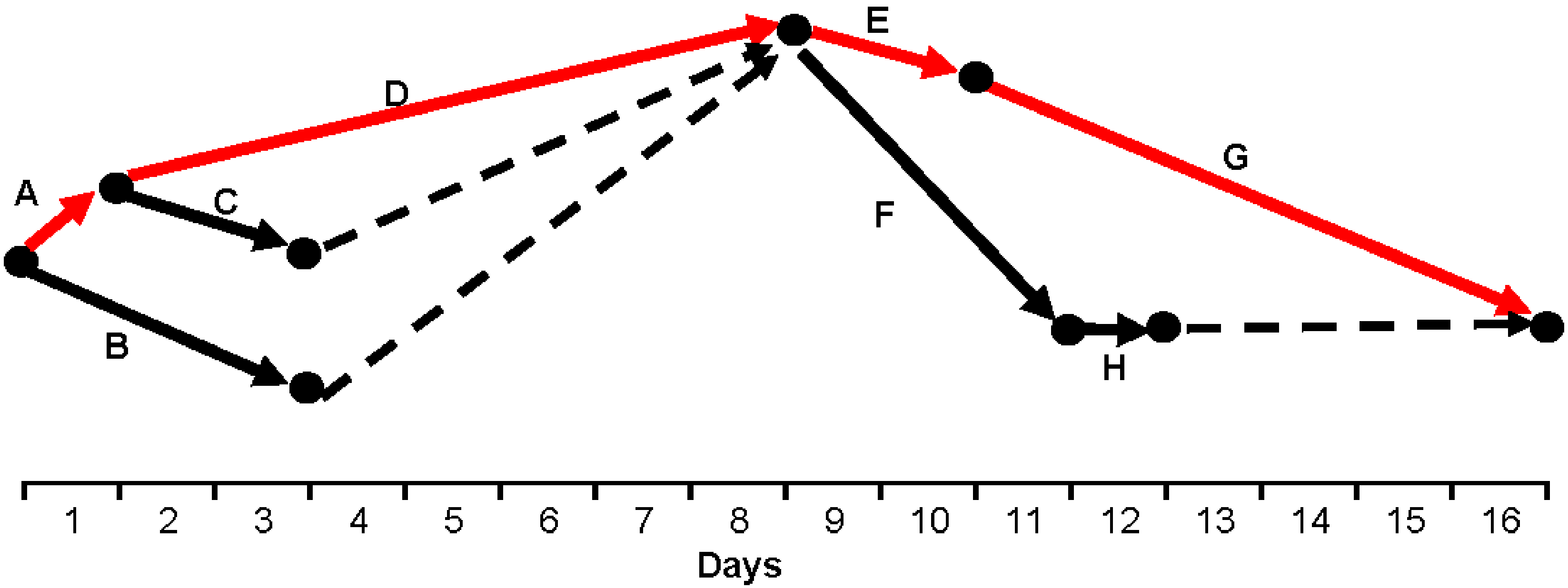
- In a project network, where do we reduce work or add resources?



- Always on the Critical Path

- Cutting work or adding resources anywhere else will have no effect on project completion

Time – Cost Tradeoff: Example



Time – Cost Tradeoff: Example

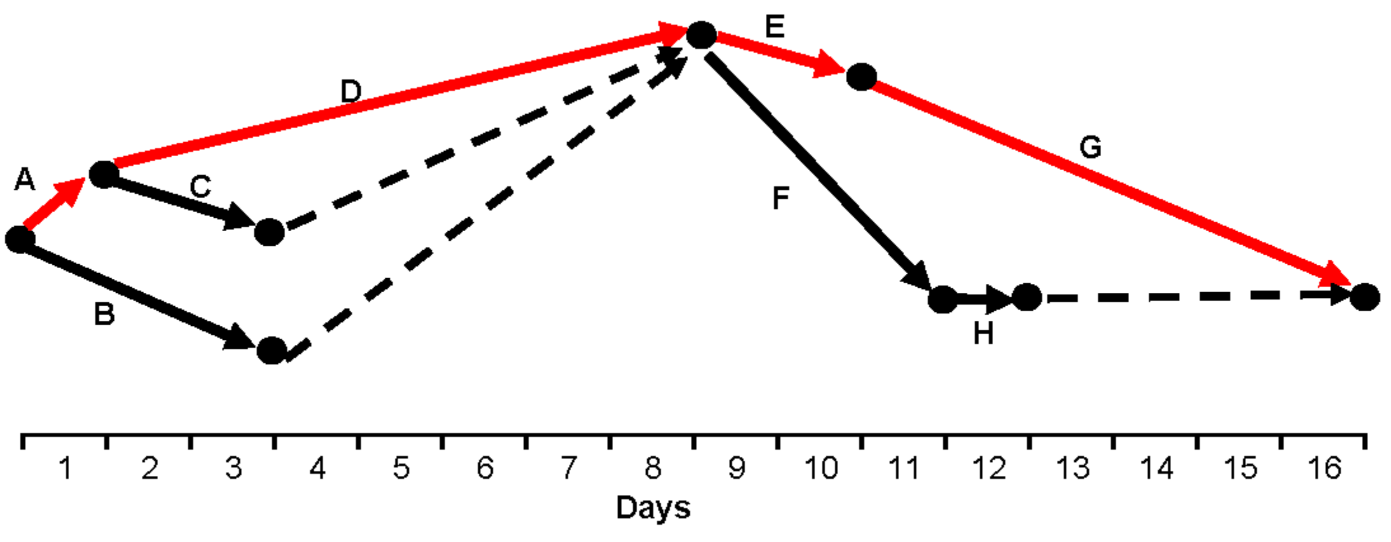
- Project completion time is estimated to be 16 days
- Suppose 16 is too long. To shorten project, must either reduce work or add resources among activities A, D, E, and G
- Suppose we cannot cut out any work, so we have to add resources. To which activity? A, D, E, or G?
- Consider the cost of these alternatives

Time – Cost Tradeoff: Example

	Normal		Crash		Cost
Activity	T_n	C_n	T_c	C_c	Slope
A	1	50	1	50	--
B	3	100	2	105	5
C	2	80	1	95	15
D	7	150	2	200	10
E	2	90	1	100	10
F	3	110	2	120	10
G	6	180	1	255	15
H	1	60	1	60	--
		820		985	

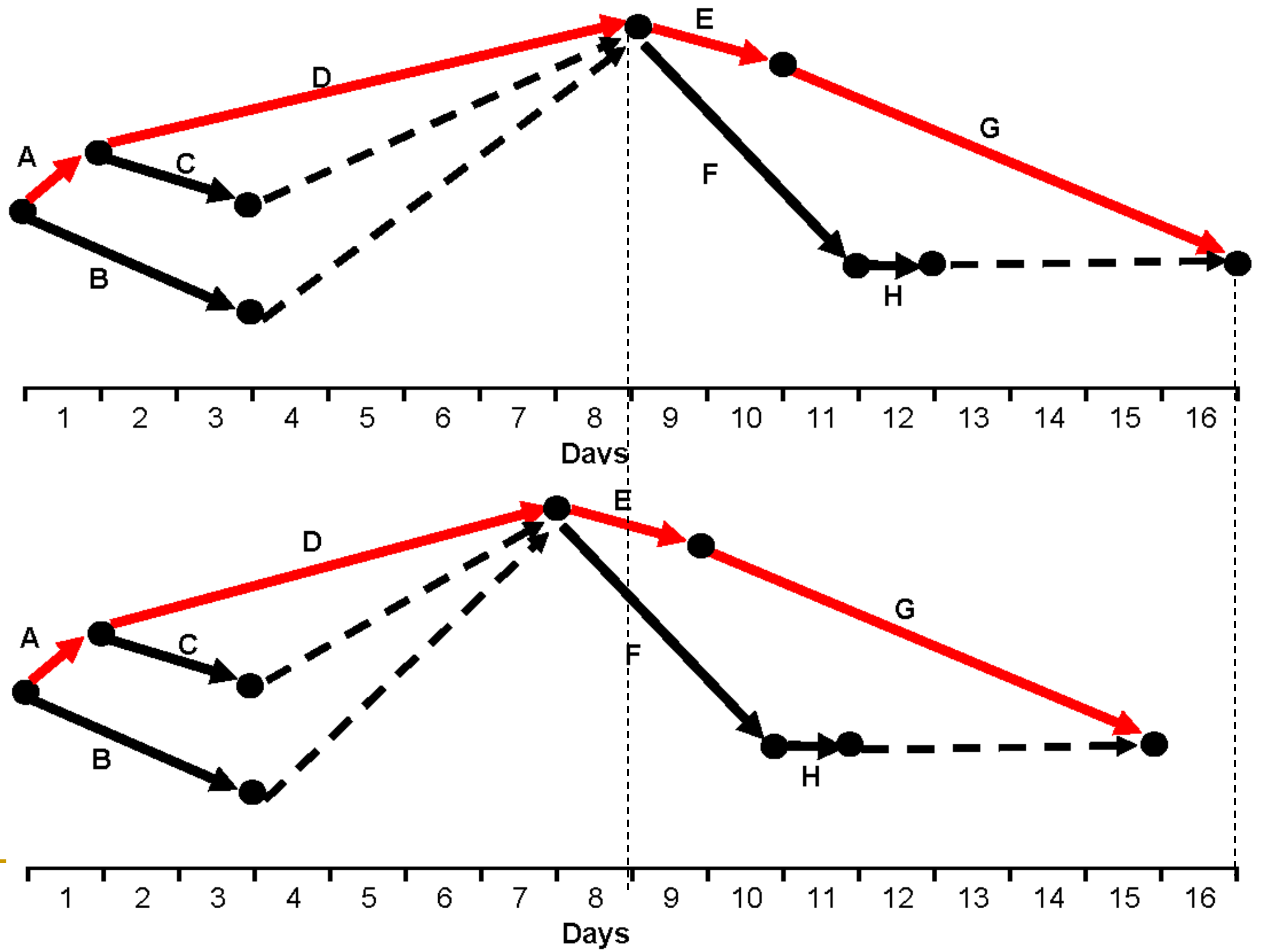
Time – Cost Tradeoff: Example

- Project takes 16 days and costs $820
- To reduce duration, add resources to activity on CP with smallest cost slope
  - Activity A cannot be reduced
  - D and E have slopes of only $10
  - Let's pick D, arbitrarily. Reduce from 7 to 6 days



Time – Cost Tradeoff: Example

Reduce duration of D from 7 to 6 days



Time – Cost Tradeoff: Example

- Reducing D by 1 day adds $10 to cost of project

- This is a “direct cost” (DC)
  - so direct cost of project increases to

DC = 820 + 10 = $830

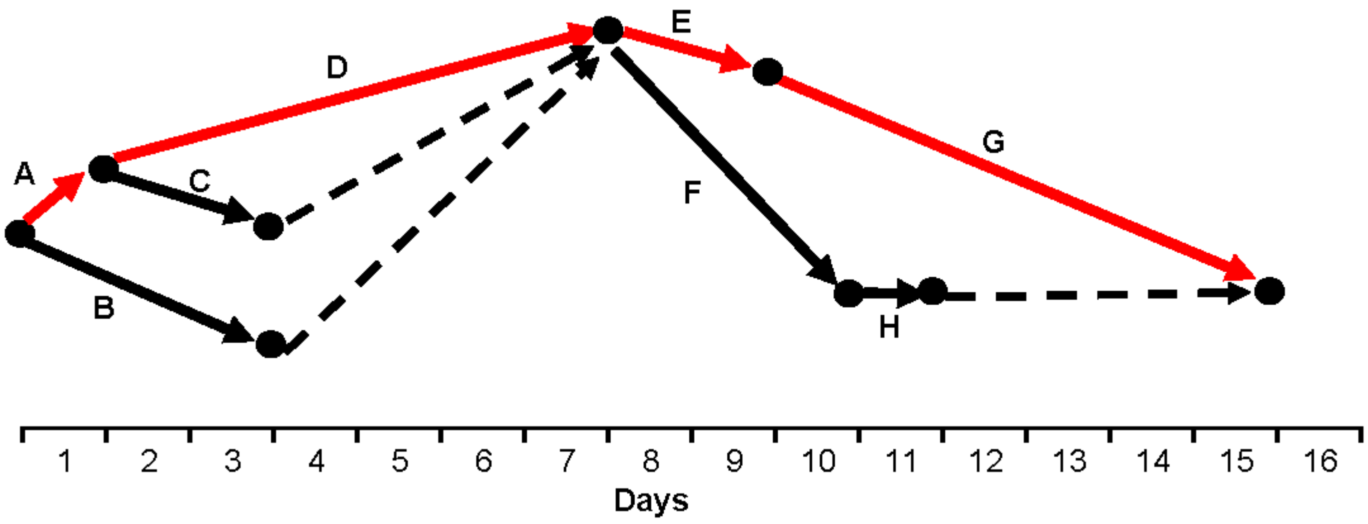
Time – Cost Tradeoff: Example

How far can an activity be reduced?

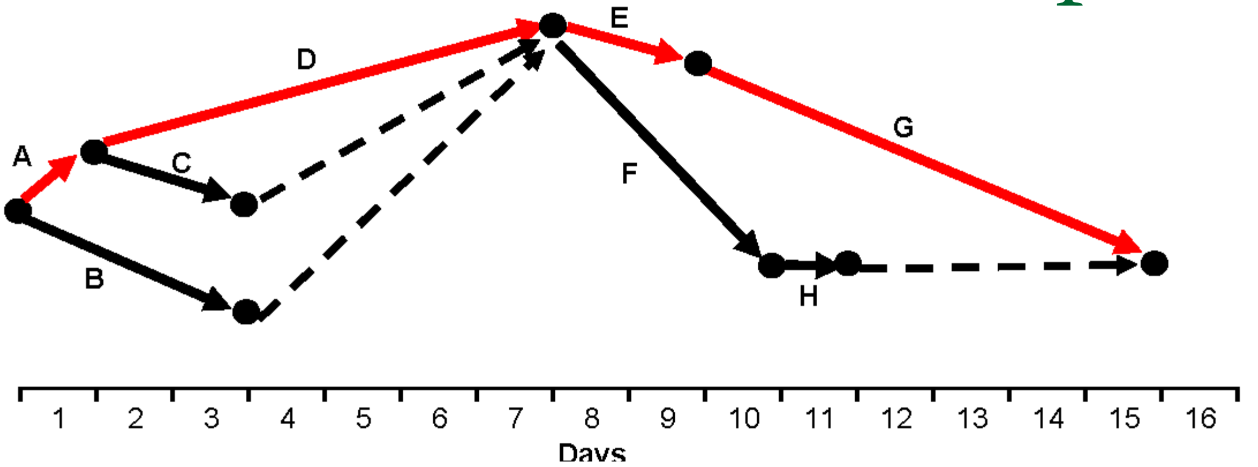
- Either to its crash time,
- or by the amount of slack time on parallel non-critical paths,
- whichever happens first

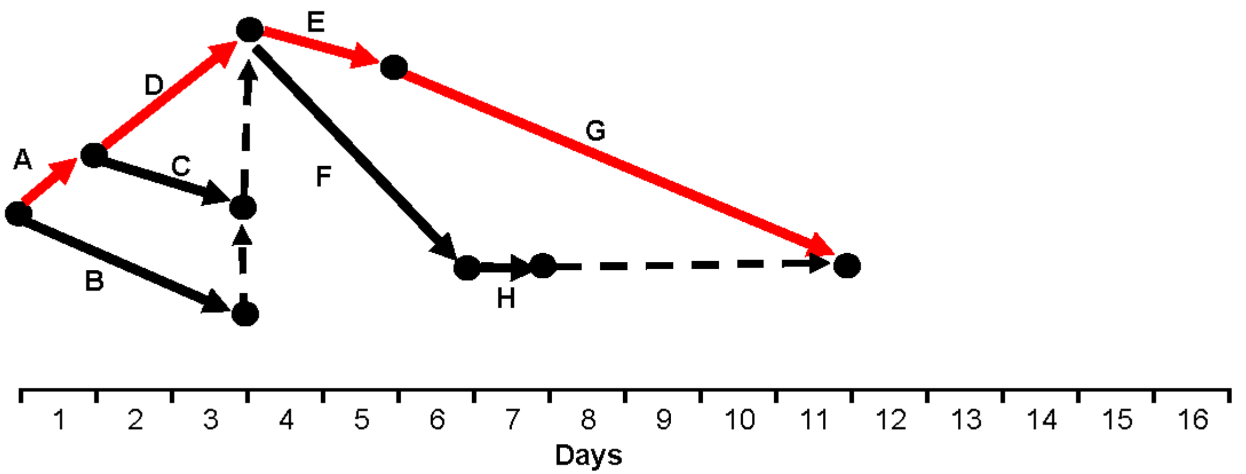
Time – Cost Tradeoff: Example

- Slack times for B and C are 5 days
- D’s crash time is 2 days
- Hence, 5 days can be taken out of D (slack time) since result is 2 days – which happens to be crash time for D



Time – Cost Tradeoff: Example





Reduce D by 5 days

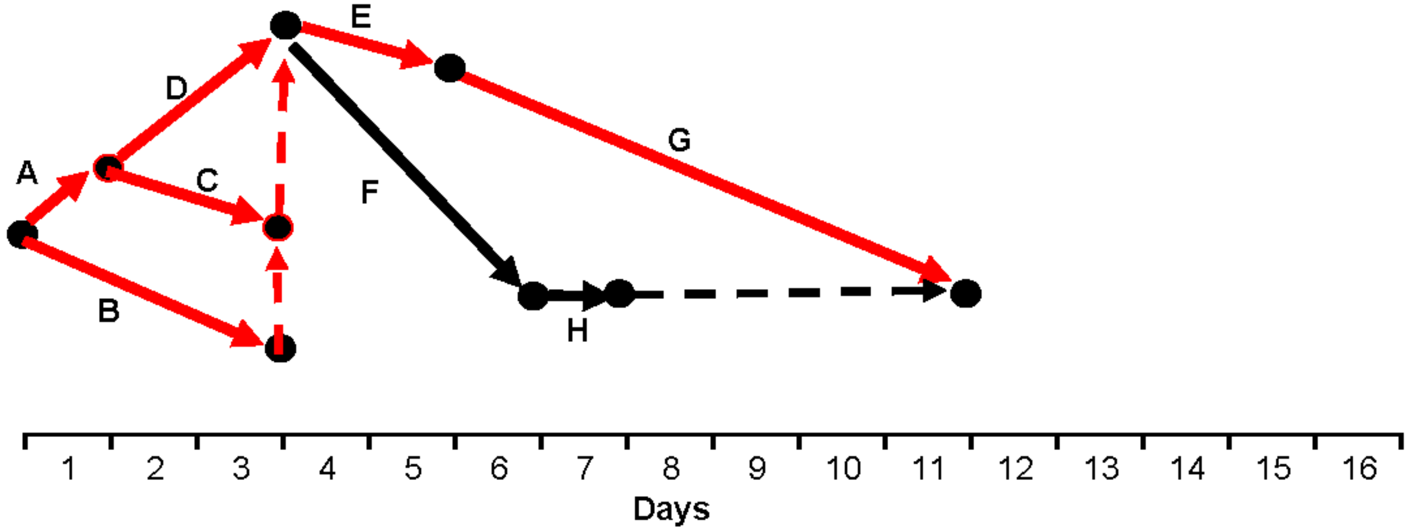
DC = 820 + 5(10) = $870

Project can be done in 11 days but will cost $870

Time – Cost Tradeoff: Example

With D now at 2 days, both C and D become critical (0 slack)

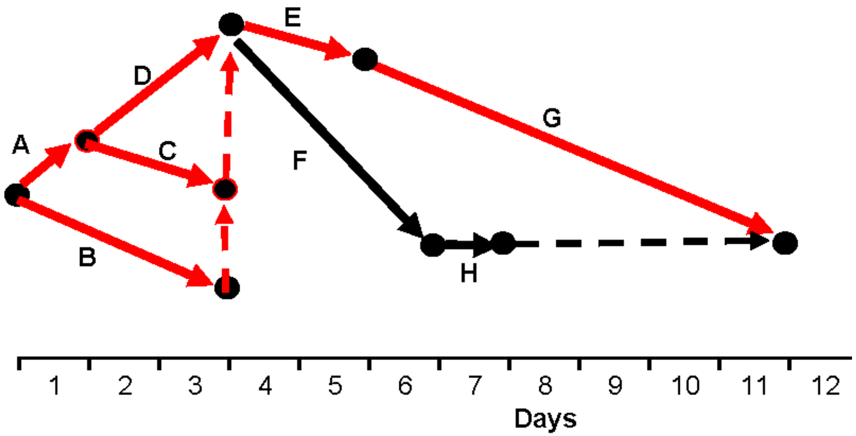
If we want to reduce project duration still more, look for remaining activities on CP with lowest cost slope. That would be E



Time – Cost Tradeoff: Example

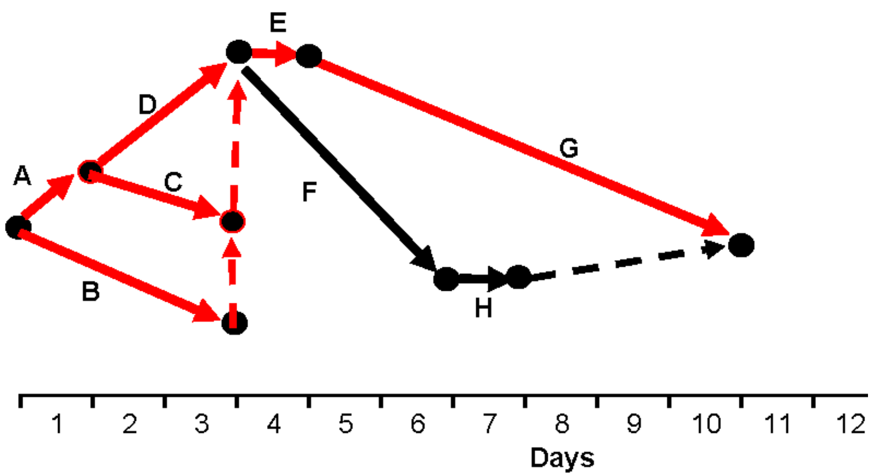
	Normal		Crash		Cost Slope	
Activity	T_n	C_n	T_c	C_c		
A	1	50	1	50	--	Cannot be crashed
B	3	100	2	105	5	Reducing without crashing A and/or D will not help
C	2	80	1	95	15	Reducing without crashing D will not help
D	7	150	2	200	10	Fully crashed
E	2	90	1	100	10	
F	3	110	2	120	10	
G	6	180	1	255	15	
H	1	60	1	60	--	

Time – Cost Tradeoff: Example



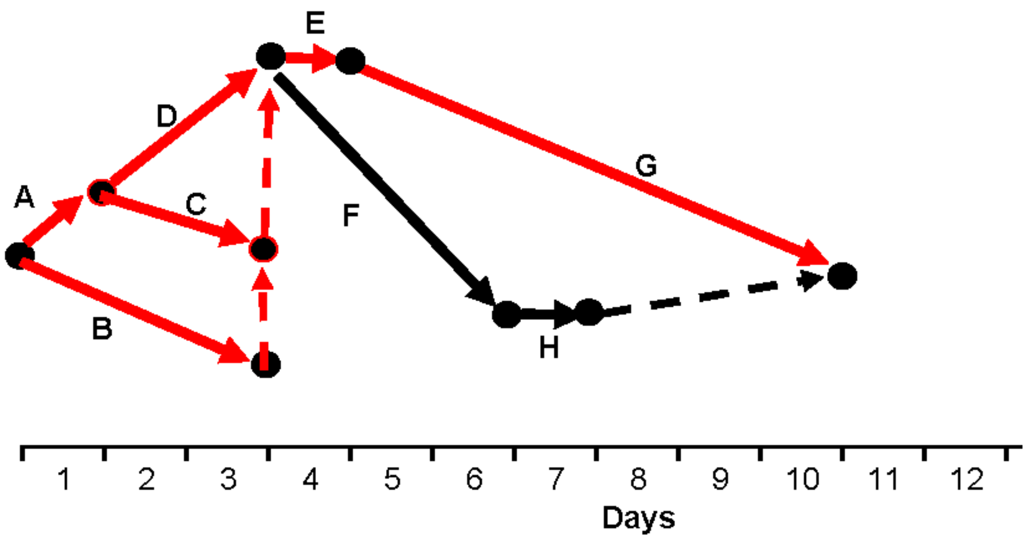
Reduce E by 1 day (from 2 days to one)

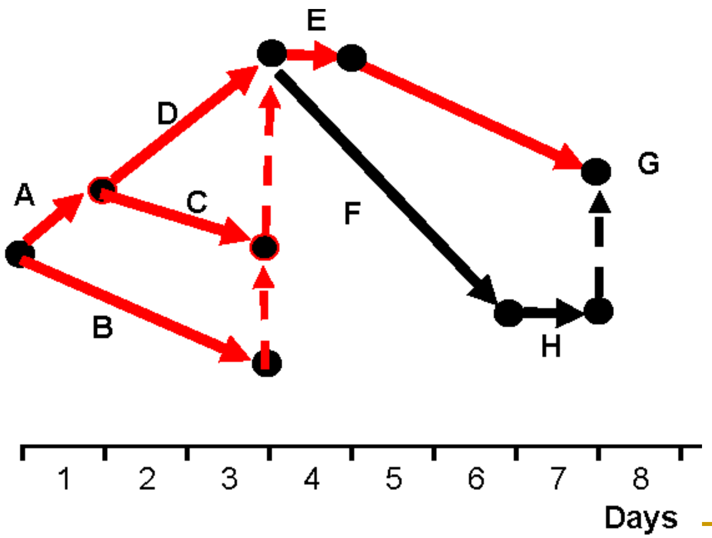
E is now fully crashed



DC = 870 + 10 = $880

Time – Cost Tradeoff: Example



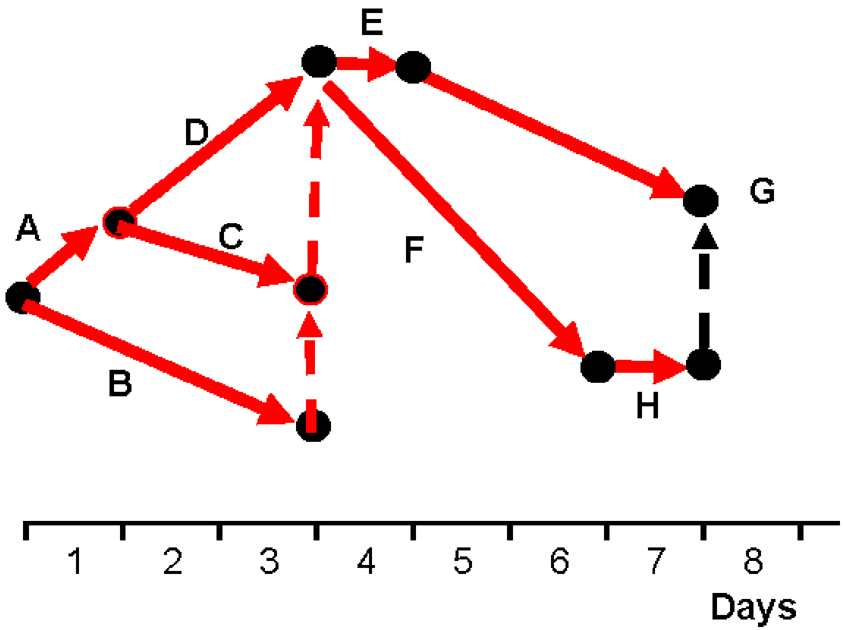


Reduce G by 3 days

DC = 880 + 3(15) = $925

Time – Cost Tradeoff: Example

At this stage, every path is critical
Here on, it will be necessary to shorten two activities



Shortening path E-G or path F-H alone will not shorten project

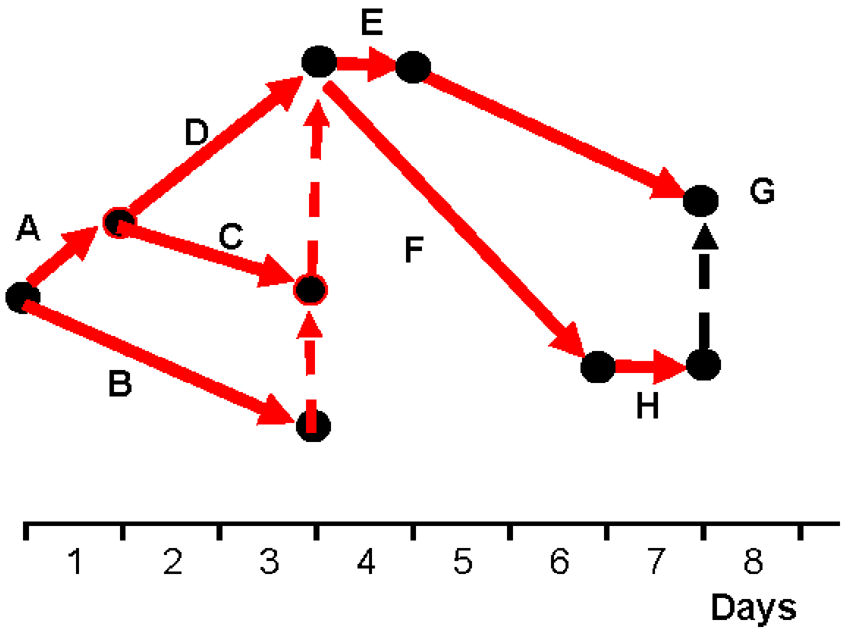
Must shorten both paths

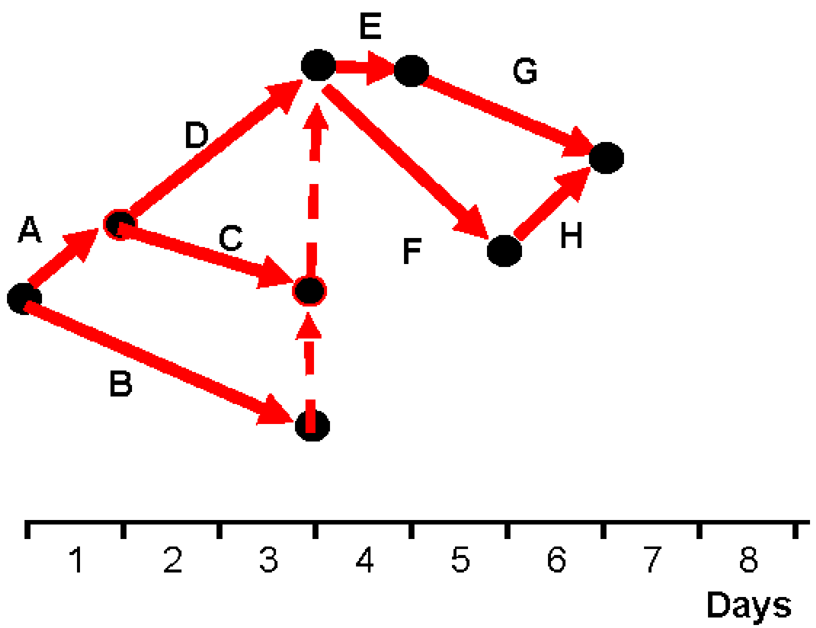
Time – Cost Tradeoff: Example

	Normal		Crash		Cost Slope	
Activity	T_n	C_n	T_c	C_c		
A	1	50	1	50	--	Cannot be crashed
B	3	100	2	105	5	Reducing without crashing A and/or D will not help
C	2	80	1	95	15	Reducing without crashing D will not help
D	7	150	2	200	10	Fully crashed
E	2	90	1	100	10	Fully crashed
F	3	110	2	120	10	
G	6	180	1	255	15	
H	1	60	1	60	--	Cannot crash

Time – Cost Tradeoff: Example

Reduce F and G by 1 day each





Time – Cost Tradeoff: Example

DC = 925 + 10 + 15 = $950

Project cannot be shortened further

Time – Cost Tradeoff: Principles

Summary of principles: Determine least expensive way to finish project by earlier date

- Focus on the critical path
- Choose least costly (lowest cost slope) alternative to shorten project duration first
- If project duration has not been reduced sufficiently, choose second-cheapest alternative, and so on
- Keep an eye on all non-critical paths
- If a non-critical path becomes critical, reduce activity duration on this path as well in next round

Time – Cost Tradeoff: Principles

In its simplest form (as illustrated in the example) this technique does not take variability – other than variability related to cost – into account

(It is a deterministic approach)

What would be the implications of we accepted that activity duration is actually variable? (As taken into account by PERT and Critical Chain methods)

Time – Cost Tradeoff: Principles

After crashing more activities (in the example discussed, *all* activities) are critical

Should any activity take longer than estimated, the whole project will be late

In practice, this implication should be taken into account

Managers can stop the process of crashing at any step to prevent further activities from becoming critical (reduce the risk of delaying the project)

Time – Cost Tradeoff: Alternative Methods

- Method 1: Crash step-by-step as illustrated
  - Start with *lowest* cost slope

- Method 2: For the *shortest* duration
  - Initially crash *all* activities
  - Relax activities
  - Relax activity with *highest* cost slope first
  - Then one with second-highest, and so on

Shortest Duration

Suppose objective of analysis is to determine *shortest time to complete* project

Method 1 (step-by-step way) gives the right answer, but

Method 2 is the faster way

Shortest Duration – Method 2

Insert crash times for *all* activities

	Normal		Crash		Cost
Activity	T_n	C_n	T_c	C_c	Slope
A	1	50	**1**	50	--
B	3	100	**2**	105	5
C	2	80	**1**	95	15
D	7	150	**2**	200	10
E	2	90	**1**	100	10
F	3	110	**2**	120	10
G	6	180	**1**	255	15
H	1	60	**1**	60	--
		820		**985**	

Shortest Duration – Method 2

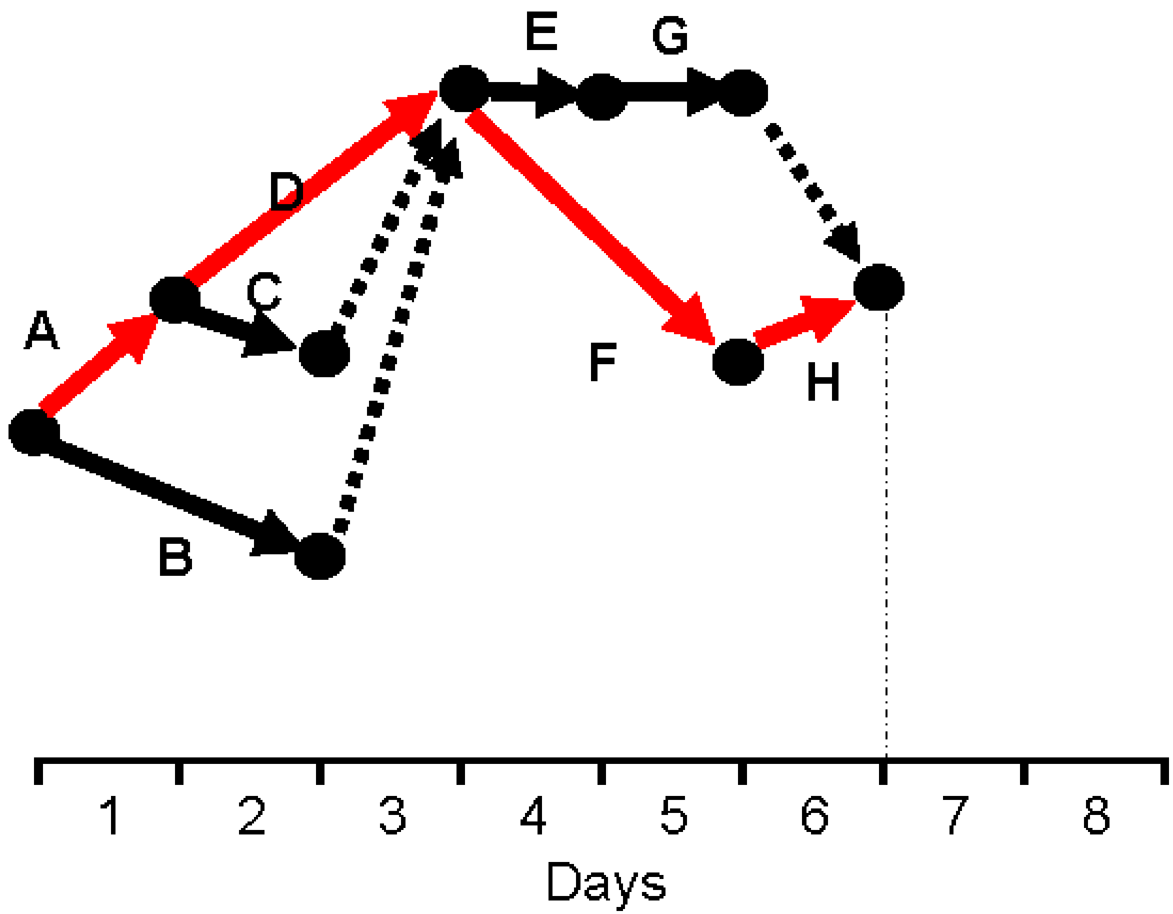
Insert crash times for *all* activities

Crash cost (crash everything):

DC = $985

Shortest Duration – Method 2

Insert crash times for *all* activities



Shortest Duration – Method 2

- This gives right answer of 6 days,
  - but at a cost of $985

- Cost is so high because everything is being crashed

- However, to complete project in 6 days it is not necessary to crash every activity,
  - Non-critical ones need not be crashed

Shortest Duration – Method 2

Non-critical activities can be “relaxed” (stretched)

Relax (stretch-out) activities that are non-critical

- either to use up their slack times, or
- until they reach their normal time

whichever happens first.

Start with non-critical activity with *highest* cost slope

Shortest Duration – Method 2

Insert crash times for *all* activities

	Normal		Crash		Cost
Activity	T_n	C_n	T_c	C_c	Slope
A	1	50	**1**	50	--
B	3	100	**2**	105	5
C	2	80	**1**	95	15
D	7	150	**2**	200	10
E	2	90	**1**	100	10
F	3	110	**2**	120	10
G	6	180	**1**	255	15
H	1	60	**1**	60	--
		820		**985**	

Shortest Duration – Method 2

Relaxing (stretching-out) activities that are non-critical

Start with non-critical activity with *highest* cost slope

G and C have highest cost slope (15)

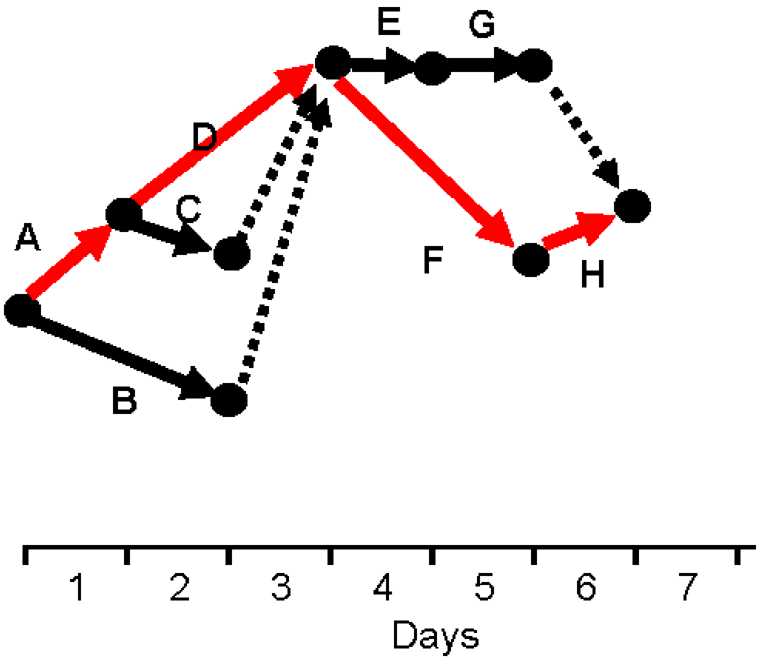
Both have one day of slack

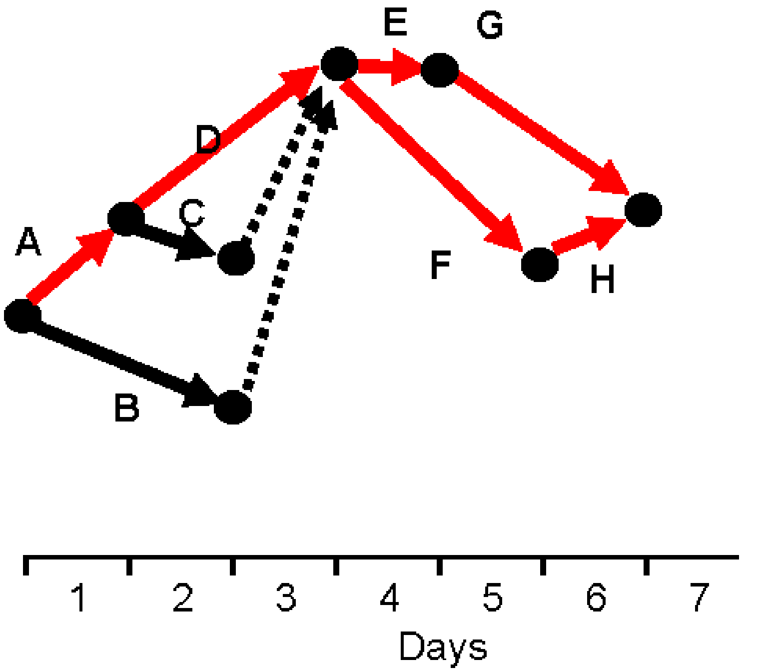
Arbitrarily choose to relax G

Shortest Duration – Method 2

Non-critical activities can be “relaxed”

Add one day to G





Shortest Duration – Method 2

Adding one day to G:

Adding time reduces cost

Hence, adding a day to G reduces its cost by $15

Effect on direct cost: DC = 985-15=$970

Shortest Duration – Method 2

Now add one day to C (C has one day slack and also has a cost slope of $15)

Adding a day to C reduces cost by a further $15

Effect on direct cost: DC =$970 – 15 = $955

Shortest Duration – Method 2

Finally add one day to B

(B has one day slack and a cost slope of only 5)

Adding a day to B reduces cost by a further $15

DC =$955 – 5 = $950

Hence,

Shortest time: 6 days

Cost to complete in shortest time: $950

Shortest Duration – Method 2

As in the case of Method 1, there is no slack and a delay of any activity would delay the project

Management can stop the process of relaxing non-critical activities at any step to prevent further activities from becoming critical (reduce the risk of delaying the project)

Duration with Least Total Cost

Longer than normal duration increases cost

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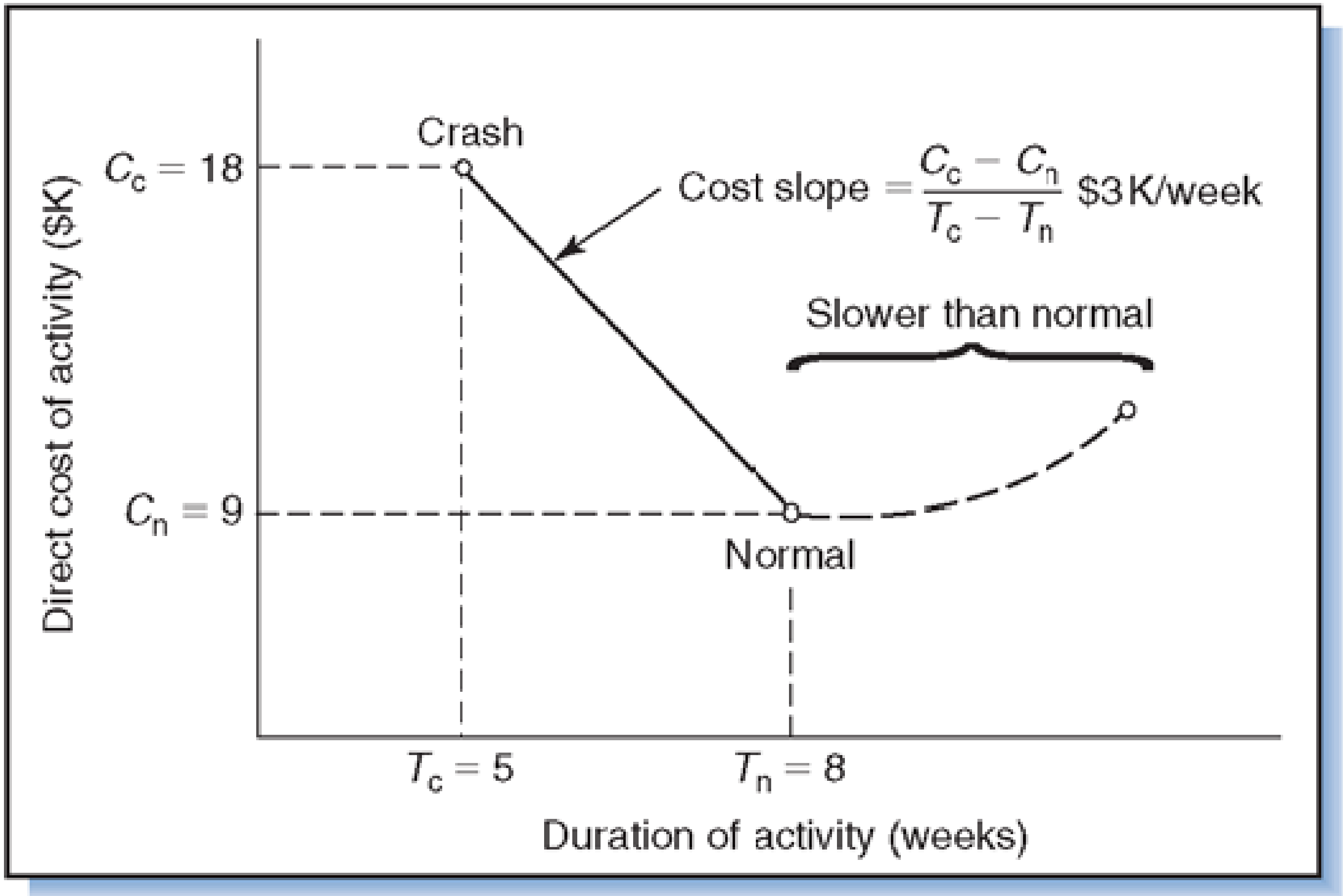


Duration with Least Total Cost

Until now we have dealt only with direct costs

Suppose objective is to complete project in whatever time results in least cost

Suppose, in addition to *direct* cost, a significant portion of project cost is "*indirect*" or overhead cost

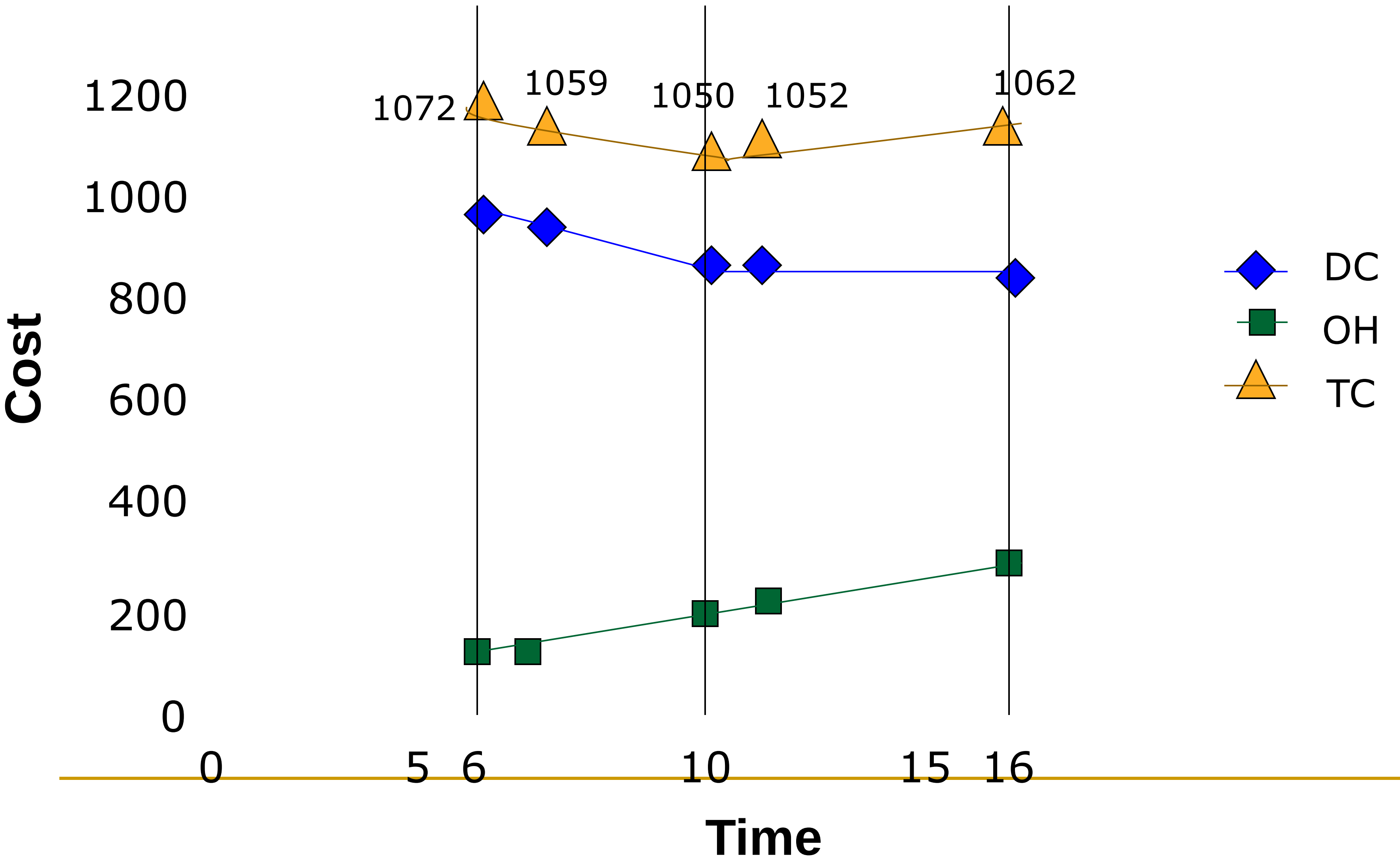
Total Project Cost (TC) = DC + Overhead (OH)

Duration with Least Total Cost

For example, suppose OH = 50 + 12(duration)

Duration	DC	OH	TC
16	820	242	1062
11	870	182	1052
10	880	170	1050
7	925	134	1059
6	950	122	1072

Duration with Least Total Cost



Least-Cost Duration (cont’d)

So, least expensive time in which to complete project is about 10 days

Second Example

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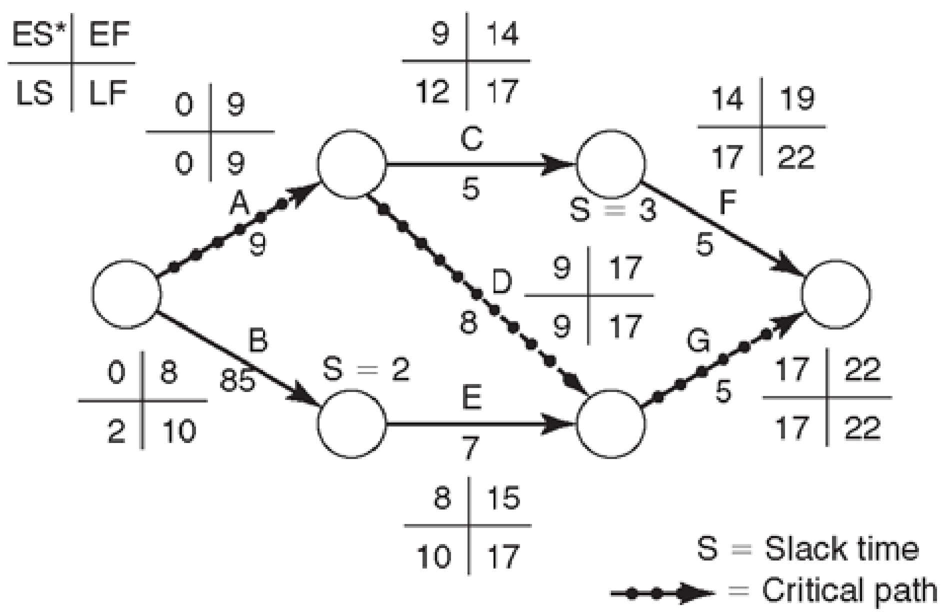
Start with all activities at normal duration

Duration: 22 weeks Cost: $55K

Shorten either A, D or G – one with smallest slope

Figure 7-3
Time–cost tradeoff for example network.



Activity	Normal T_n	Normal C_n	Crash T_c	Crash C_c	Cost Slope
A	9	10	6	16	2
B	8	9	5	18	3
C	5	7	4	8	1
D	8	9	6	19	5
E	7	7	3	15	2
F	5	5	5	5	
G	5	8	2	23	5
		$55K		$104K	

*Note: Assume start occurs *immediately* upon the finish of all immediate predecessors.

Second Example (Cont'd)

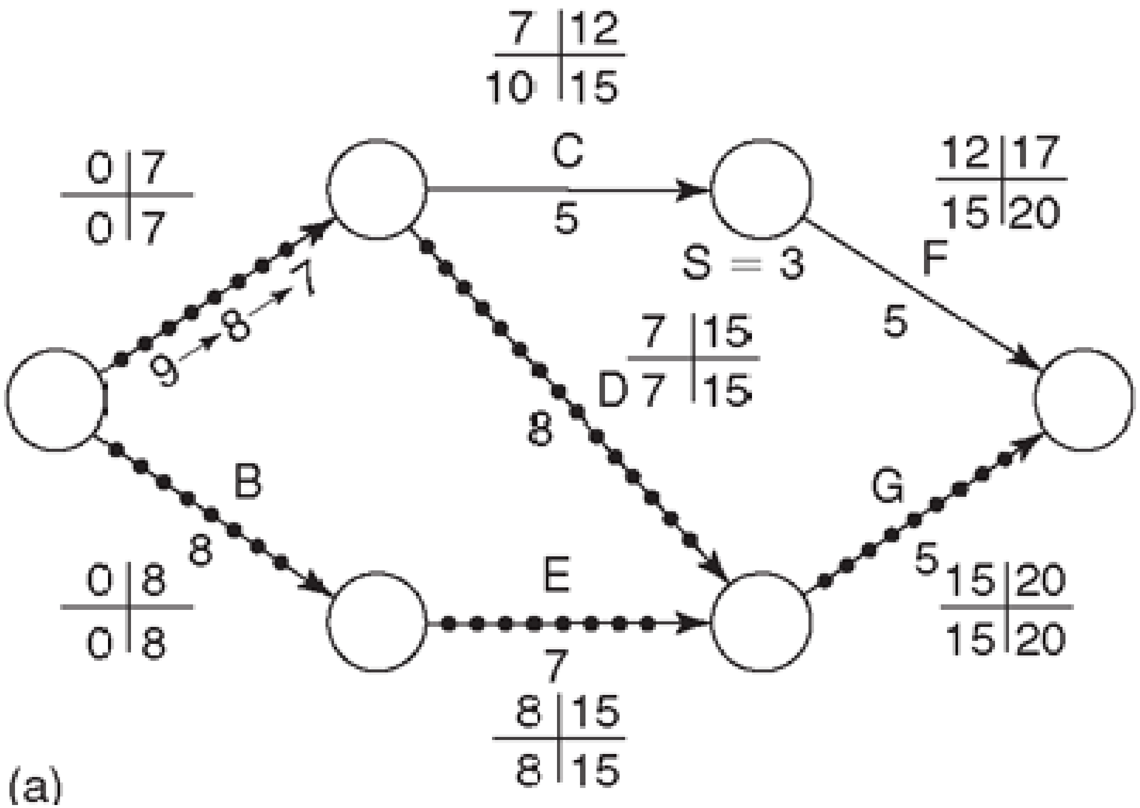
Reducing A by 1 week reduces project duration to 21 weeks and add $2K
Project cost now $57K

Cut another week from A:
Project duration: 20 weeks
Project cost: $59K

Path BE now also becomes critical

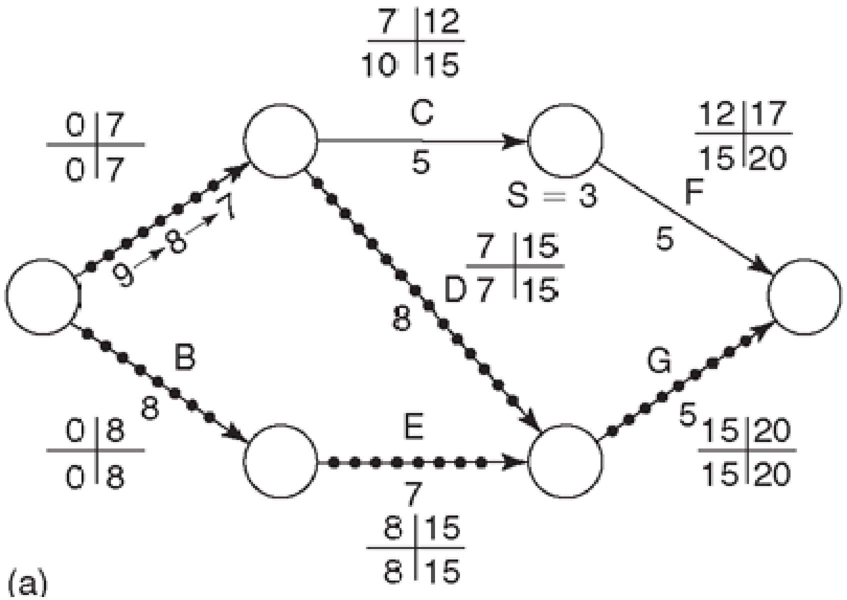
Second Example (Cont'd)

Path BE now also critical



Both ADG and BEG now need to be shortened

Second Example (Cont'd)



Activity	Normal T_n	Normal C_n	Crash T_c	Crash C_c	Cost Slope
A	9	10	6	16	2
B	8	9	5	18	3
C	5	7	4	8	1
D	8	9	6	19	5
E	7	7	3	15	2
F	5	5	5	5	
G	5	8	2	23	5
		$55K		$104K	

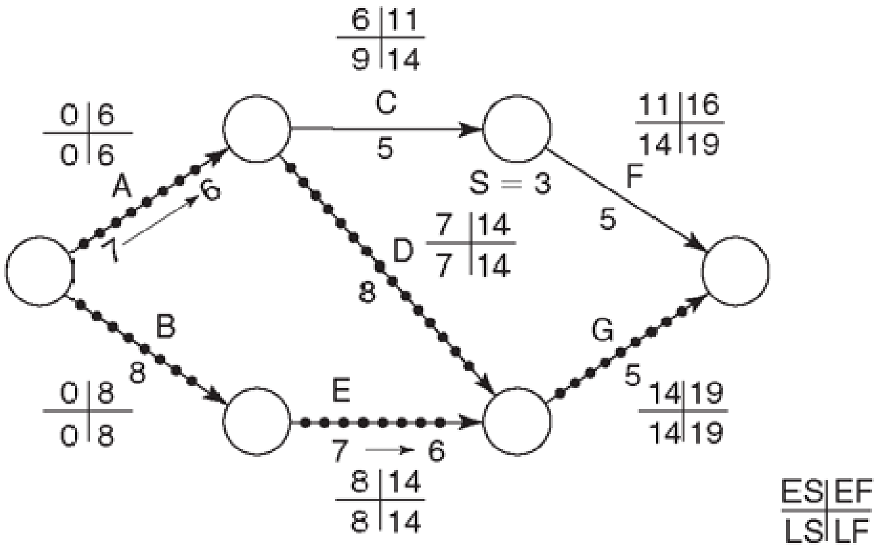
Both ADG and BEG need to be shortened

Least costly: Reduce both A and E

Project duration: 19 weeks

Cost = $59K + $2K + $2K = $63K

Second Example (Cont'd)



(b) (after reducing times)

Activity	Normal T_n	C_n	Crash T_c	C_c	Cost Slope
A	9	10	6	16	2
B	8	9	5	18	3
C	5	7	4	8	1
D	8	9	6	19	5
E	7	7	3	15	2
F	5	5	5	5	
G	5	8	2	23	5
		$55K		$104K	

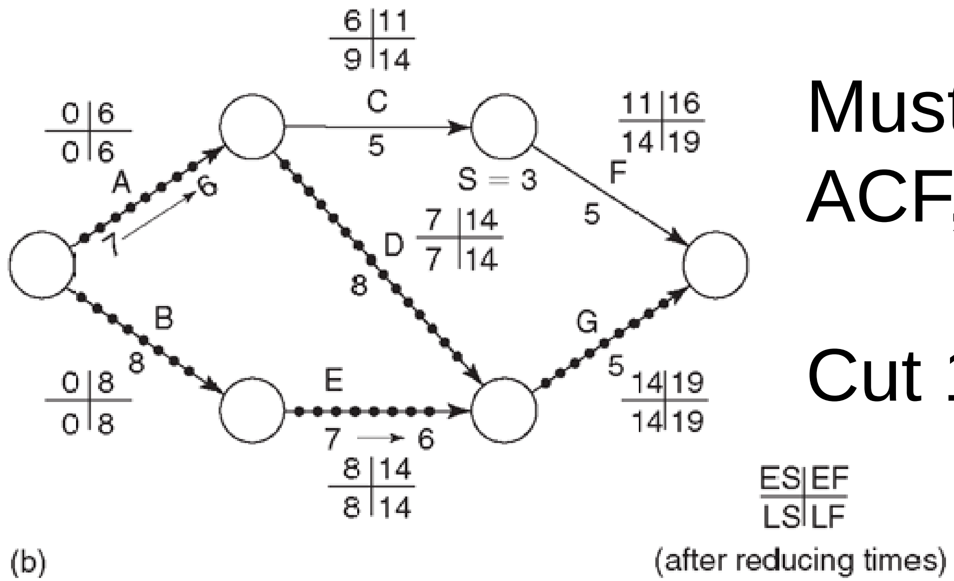
Duration of A is now 6 weeks – fully crashed

Least costly way now to reduce G

G can be reduced by 3 weeks (5 – 2)
This adds a cost of $5K x 3

Second Example (Cont’d)

Reducing G to 2 weeks causes CF to get critical



Must now reduce all three: ACF, ADG and BEG

Cut 1 week from E, D and C

Duration: 15 weeks

Cost: $86K

Second Example (Cont'd)

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Summary

Table 7-1 Duration reduction and associated cost increase.

Step	Duration (T_E, weeks)	Activities on CP with Least Cost Slope	Cost of Project (K$)
1*	22		$55
2	21	A ($2)	$55 + $2 = $57
3	20	A ($2)	$57 + $2 = $59
4	19	A ($2), E ($2)	$59 + $2 + $2 = $63
5, 6, 7	18, 17, 16	G ($5)	$63 + $5 + $5 + $5 = $78
8	15	E ($2), D ($5), C ($1)	$78 + $2 + $5 + $1 = $86

*Duration and cost using normal conditions.

Second Example – Method 2

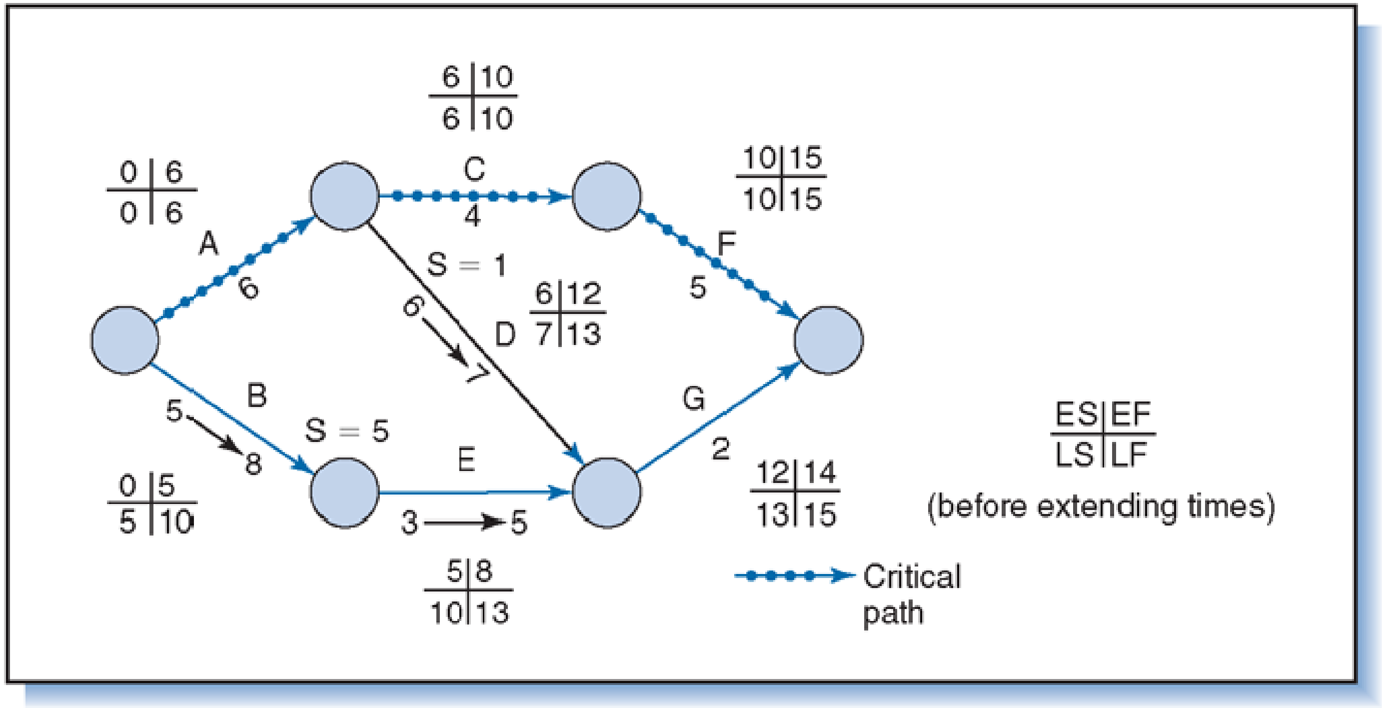
- Suppose objective of analysis is to determine *shortest time to complete* project

- **Method 2** - the faster way

Second Example – Method 2

Crash all activities at once

Figure 7-5
Example network using crash times.



Second Example – Method 2

Crash all activities at once

	Normal		Crash		
Activity	T_n	C_n	T_c	C_c	Cost Slope
A	9	10	6	16	2
B	8	9	5	18	3
C	5	7	4	8	1
D	8	9	6	19	5
E	7	7	3	15	2
F	5	5	5	5	
G	5	8	2	23	5
		\$55K		\$104K	

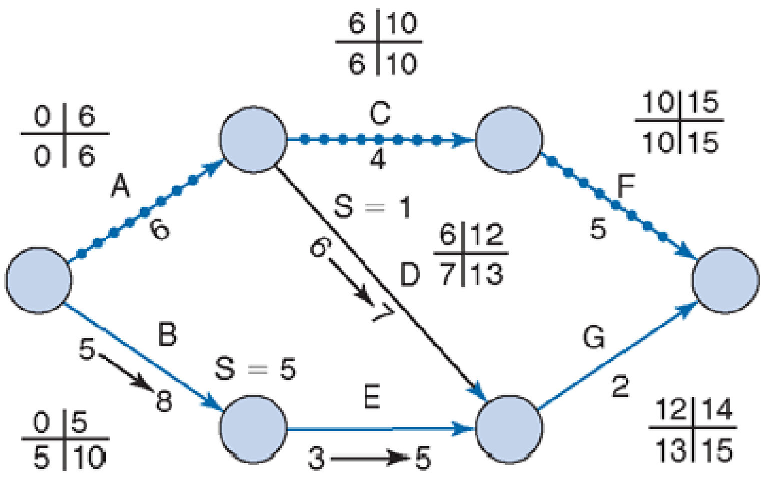
This leads to a cost of \$104K

Second Example – Method 2

- Cost of $104K can be reduced
- Do this by stretching non-critical activities
- Start stretching activities with *highest* slope to get maximum reduction first

Second Example – Method 2

Stretch activities with highest slope first



Activity	Normal T_n	Normal C_n	Crash T_c	Crash C_c	Cost Slope
A	9	10	6	16	2
B	8	9	5	18	3
C	5	7	4	8	1
D	8	9	6	19	5
E	7	7	3	15	2
F	5	5	5	5	
G	5	8	2	23	5
		\$55K		\$104K	

Path BEG has 5 weeks slack

B can be stretched by 3 weeks

Stretch E by 2 weeks and D by one

Second Example – Method 2

Activity	Normal T_n	Normal C_n	Crash T_c	Crash C_c	Cost Slope
A	9	10	6	16	2
B	8	9	5	18	3
C	5	7	4	8	1
D	8	9	6	19	5
E	7	7	3	15	2
F	5	5	5	5	
G	5	8	2	23	5
		$55K		$104K	

Stretching B by 3 weeks saves 3 x $3K = $9K

Stretching E by 2 weeks saves 2 x $2K = $4K

Stretching D by one week saves $ 5K

Cost = $104K - $9K - $4K - $5K = $86K

Second Example – Method 2

Total Cost = Direct Cost + Indirect Cost

Suppose:

Indirect Cost = \$10K + \$3K (T_e)

Where T_e = expected duration (weeks)

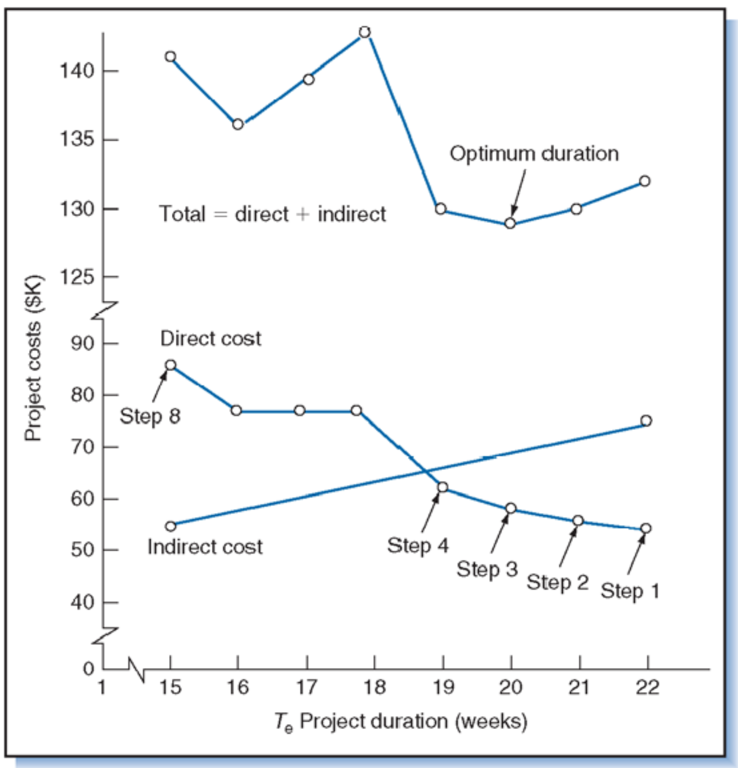


Figure 7-6
Total time–cost tradeoff for the project.

Second Example (Cont'd)

Suppose the due date is week 18 with a $2K/week bonus for early finish and a $1K/week penalty for finishing late

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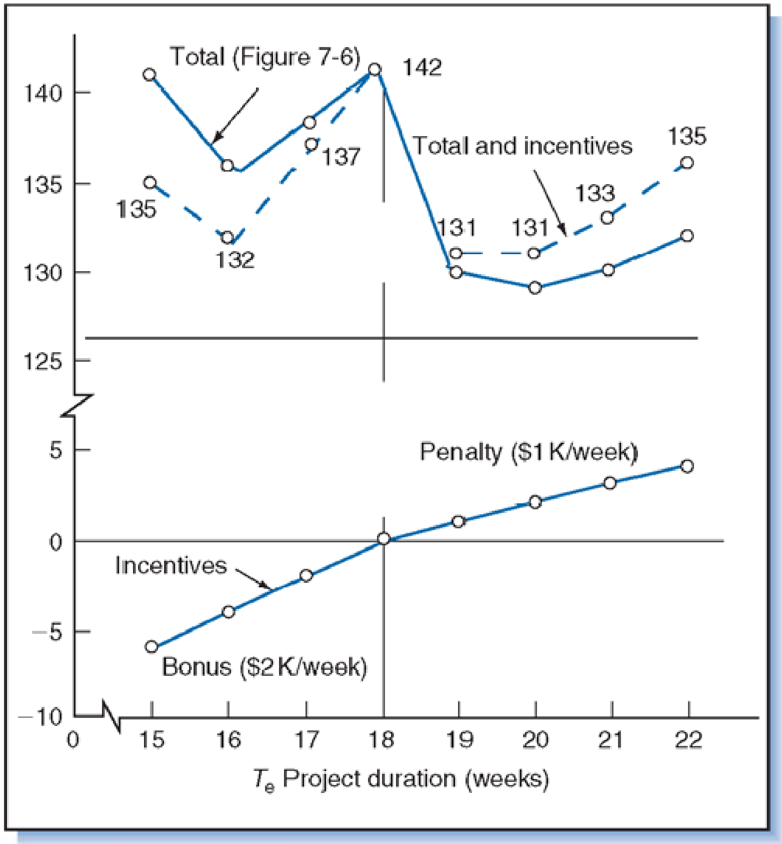

Figure 7-7
Time–cost tradeoff for the project with incentives.

Practical Matters

It is good “selling” to propose two alternatives rather than only one

A choice between Plan A and Plan B, rather than between “the plan” or nothing (“take it or leave it”)

- E.g. the board of a petrochemical company has a choice:
  - A more expensive plant, in operation earlier
  - A less expensive plant, in operation only later

Practical Matters

The practice of crashing until ***all*** activities become critical (or, with Method 2, relaxing activities until ***all*** activities become critical) is not practical: if all activities are critical and any one activity takes longer than planned, the project will be late.

Management should check the step-wise process and decide at which stage this risk becomes unacceptable – the CPM technique is *not* a mechanistic process

Practical Matters

For projects in developing countries, a longer duration (at a higher cost) is sometimes preferred in order to create work for unemployed people in the relevant community

It is claimed that this also creates buy-in

This is obviously *not* applicable to any project in a competitive environment

Practical Matters

- Project managers “know” which activities are more costly, which are less
- Some prefer to use this knowledge rather than crash-normal times to determine which activities to speed up or relax – nonetheless, the CPM technique provides valuable insight in the trade-off between cost and time